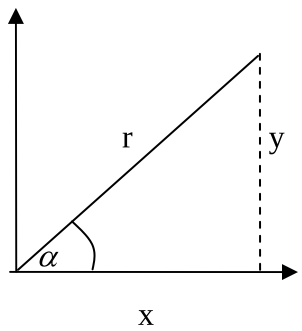
TRIGONOMETRI

Pengertian Sinus, Cosinus dan Tangen



$\text{Sin } \alpha = \frac{y}{r}$

$\text{Cos } \alpha = \frac{x}{r}$

$\text{Tan } \alpha = \frac{y}{x}$

Hubungan Fungsi Trigonometri :

1. $\sin^2 \alpha + \cos^2 \alpha = 1$

2. $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$

3. $\sec \alpha = \frac{1}{\cos \alpha}$

4. $\text{cosec } \alpha = \frac{1}{\sin \alpha}$

5 . $\text{cotan } \alpha = \frac{\cos \alpha}{\sin \alpha}$

6. $\tan^2 \alpha + 1 = \sec^2 \alpha \quad \Rightarrow \sin^2 \alpha + \cos^2 \alpha = 1$

$$\Rightarrow \frac{\sin^2 \alpha}{\cos^2 \alpha} + \frac{\cos^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha}$$

$$\Rightarrow \tan^2 \alpha + 1 = \sec^2 \alpha \rightarrow \text{bukti}$$

7. $\cot an^2\alpha + 1 = \cos ec^2\alpha \quad \Rightarrow \sin^2\alpha + \cos^2\alpha = 1$

$$\Rightarrow \frac{\sin^2\alpha}{\sin^2\alpha} + \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha}$$

$$\Rightarrow \frac{\sin^2\alpha}{\sin^2\alpha} + \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha}$$

$$\Rightarrow 1 + \cot an^2\alpha = \cos ec^2\alpha$$ → bukti

Rumus-rumus Penjumlahan dan Pengurangan :

1. sin (A + B) = sin A cos B + cos A Sin B
2. sin (A - B) = sin A cos B - cos A Sin B
3. cos (A + B) = cos A cos B – sin A Sin B
4. cos (A - B) = cos A cos B + sin A Sin B
5. tan (A + B) = $\dfrac{\tan A + \tan B}{1 - \tan A.\tan B}$
6. tan (A - B) = $\dfrac{\tan A - \tan B}{1 + \tan A.\tan B}$

Rumus-rumus Sudut Rangkap :

1. sin 2A = 2 sin A cosA
2. cos 2A = $\cos^2 A - \sin^2 A$ (ingat : $\sin^2 A + \cos^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A$
   $\Rightarrow \cos^2 A = 1 - \sin^2 A$)
   kalau dimasukkan ke dalam rumus maka :

   $= 1 - 2\sin^2 A \Leftrightarrow \cos^2 A - \sin^2 A = (1 - \sin^2 A) - \sin^2 A = 1 - \sin^2 A - \sin^2 A$
   $= 1 - 2\sin^2 A$

   $= 2\cos^2 A - 1 \Leftrightarrow$ dengan cara yang sama bias dibuktikan
3. tan 2A = $\dfrac{2\tan A}{1 - (\tan A)^2}$

Rumus Jumlah Fungsi :

Perkalian → jumlah/selisih

1. 2 sin A cos B = sin (A+B) + sin (A-B)
2. 2 cos A sin B = sin (A+B) – sin (A-B)
3. 2 cos A cos B= cos (A+B) + cos (A-B)
4. -2sin A sin B = cos (A+B) – cos(A-B)

Jumlah/selisih → perkalian

1. $\text{Sin } A + \sin B = 2 \sin \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

2. $\text{Sin } A - \sin B = 2 \cos \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$

3. $\cos A + \cos B = 2 \cos \frac{1}{2} (A + B) \cos \frac{1}{2} (A - B)$

4. $\cos A - \cos B = -2 \sin \frac{1}{2} (A + B) \sin \frac{1}{2} (A - B)$

Sudut-sudut istimewa :

α	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
Cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	~

Tanda-tanda fungsi pada setiap kuadrant :

	Kuadrant I α	Kuadrant II $180^0 - \alpha$	Kuadrant III $180^0 + \alpha$	Kuadrant IV $360^0 - \alpha$
Sin	+	+	-	-
Cos	+	-	-	+
Tan	+	-	+	-

Rumus-rumus Sudut :

- **Sudut** 180^0 **-** α **dan** α **(Kuadran kedua)**

  sin (180^0 - α) = sin α
  cos (180^0 - α) = - cos α
  tan (180^0 - α) = - tan α
  cosec (180^0 - α) = cosec α
  sec (180^0 - α) = - sec α
  cotan (180^0 - α) = - cotan α

- **Sudut** 180^0 **+** α **dan** α **(Kuadran ketiga)**

  sin (180^0 + α) = - sin α
  cos (180^0 + α) = - cos α
  tan (180^0 + α) = tan α
  cosec (180^0 + α) = - cosec α
  sec (180^0 + α) = - sec α
  cotan (180^0 + α) = cotan α

- **Sudut** 360^0 **-** α **dan** α **(Kuadran keempat)**

  sin (360^0 - α) = - sin α
  cos (360^0 - α) = cos α
  tan (360^0 - α) = - tan α
  cosec (360^0 - α) = - cosec α
  sec (360^0 - α) = sec α
  cotan (360^0 - α) = - cotan α

- **Sudut** 360^0 **+** α **dan** α **(Kuadran pertama)**

  sin (360^0 + α) = sin α
  cos (360^0 + α) = cos α
  tan (360^0 + α) = tan α
  cosec (360^0 + α) = cosec α
  sec (360^0 + α) = sec α
  cotan (360^0 + α) = cotan α

- **Sudut -α dan α**

  sin (-α) = -sin α
  cos (-α) = cos α
  tan (-α) = -tan α
  cosec (-α) = -cosec α
  sec (-α) = sec α
  cotan (-α) = -cotan α

- **Sudut (90^0 - α) dan α (Kuadran pertama)**

  sin (90^0 - α) = cos α
  cos (90^0 - α) = sin α
  tan (90^0 - α) = cotan α
  cot (90^0 - α) = tan α
  sec (90^0 - α) = cosec α
  cosec (90^0 - α) = sec α

- **Sudut (90^0 + α) dan α (Kuadran kedua)**

  sin (90^0 + α) = cos α
  cos (90^0 + α) = -sin α
  tan (90^0 + α) = -cotan α
  cot (90^0 + α) = =tan α
  sec (90^0 + α) = -cosec α
  cosec (90^0 + α) = sec α

- **Sudut (270^0 - α) dan α (Kuadran ketiga)**

  sin (270^0 - α) = -cos α
  cos (270^0 - α) = -sin α
  tan (270^0 - α) = cotan α
  cot (270^0 - α) = tan α
  sec (270^0 - α) = -cosec α
  cosec (270^0 - α) = sec α

- **Sudut (270^0 + α) dan α (Kuadran keempat)**

  sin (270^0 + α) = -cos α
  cos (270^0 + α) = sin α
  tan (270^0 + α) = -cotan α

$\cot(270^0 + \alpha) = -\tan\alpha$
$\sec(270^0 + \alpha) = \text{cosec}\ \alpha$
$\text{cosec}(270^0 + \alpha) = -\sec\ \alpha$

- **Sudut yang melebihi satu putaran penuh :**

$\sin(k.360^0 + \alpha) = \sin\ \alpha$
$\cos(k.360^0 + \alpha) = \cos\ \alpha$
$\tan(k.360^0 + \alpha) = \tan\ \alpha$
$\text{cosec}(k.360^0 + \alpha) = \text{cosec}\ \alpha$
$\sec(k.360^0 + \alpha) = \sec\ \alpha$
$\text{cotan}(k.360^0 + \alpha) = \text{cotan}\ \alpha$

dengan k bilangan bulat

Persamaan dan pertidaksamaan Trigonometri

1. Persamaan

Rumus umum penyelesaian persamaan trigonometri adalah :

* $\sin x = \sin\ \alpha$, maka $x_1 = \alpha + k.360^0$
$x_2 = (180^0 - \alpha) + k.360^0$

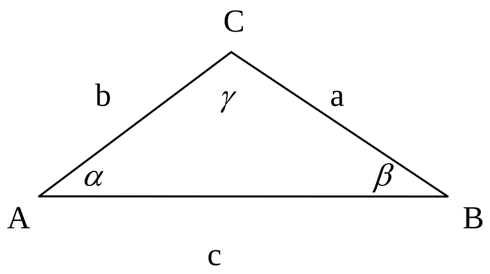
* $\cos x = \cos\ \alpha$, maka $x_{1,2} = \pm\ \alpha + k.360^0$

* $\tan x = \tan\ \alpha$, maka $x = \alpha + k.\ 180^0$

2. Pertidaksamaan

Pertidaksamaan-pertidaksamaan trigonometri seperti $\sin ax \leq c$, $\cos ax \geq c$ dan sebagainya dapat diselesaiakan dengan menggunakan langkah-langkah umum pertidaksamaan seperti :
- Diagram garis bilangan
- Grafik fungsi trigonometri

Aturan sinus dan cosinus



aturan sinus

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Aturan cosinus

1. $a^2 = b^2 + c^2$ - 2bc cos α

2. $b^2 = a^2 + c^2$ - 2ac cos β

3. $c^2 = a^2 + b^2$ - 2ab cos γ

Luas Segitiga

Luas segitiga = $\frac{1}{2}$ ab sin γ

= $\frac{1}{2}$ ac sin β

= $\frac{1}{2}$ bc sin α

Nilai Maksimum dan Minimum

1. Jika y = k cos (x + nπ) dengan k > 0 maka

a. maksimum jika y = k dimana cos (x + nπ) = 1 sehingga (x + nπ)= 0
b. minimum jika y = -k dimana cos (x + nπ) = -1 sehingga (x + nπ)= π

2. Jika $y = k \sin (x + n\pi)$ dengan $k > 0$ maka

a. maksimum jika $y = k$ dimana $\sin (x + n\pi) = 1$ sehingga $(x + n\pi) = \frac{\pi}{2}$

b. minimum jika $y = -k$ dimana $\sin (x + n\pi) = -1$ sehingga $(x + n\pi) = \frac{3\pi}{2}$

Contoh-contoh soal dan Pembahasan → baca di postingan berikutnya……….